Harold Daw Flame Table

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Abstract

A two-dimensional flame table was built to demonstrate the eigen modes of a wave in a cavity. The table is based on Bernoulli's principle and can be related to the less complex Ruben's tube. Expected images of various frequencies were created using Mathematica, and then compared to actual photographs of the various states. In depth theory behind the physics of the table is described as well as an in depth description of the apparatus. A commentary on mistakes made in design are also included in this paper.

Introduction

Combined, the curiosity of fire and the beauty of musical notes generates a dramatic visual phenomena. The Rubens flame tube is the most well-known apparatus for demonstrating such phenomena. The standing waves produced by a given frequency cause flames coming from along the tube to change in amplitude, depending on the pressure nodes [8]. Similar phenomena involving standing waves include Chladni plates and Kundt tubes. Chladni plates consist of a plane surface covered in a fine sand. A speaker is placed below the plate and eigen modes are displayed due to movement of the sand[7]. Kundt tubes consist of fine cork dust inside of a tube that is driven by a speaker at one end. The Kundt tube is analogous to the Rubens flame tube in its investigation of standing waves [2]. The most visually striking of visual sound wave phenomena is the Harold Daw flame table.

The Daw flame table is analogous to the aforementioned Chaldni plate in that it displays eigen modes. The flame table specifically displays the eigen modes of a two dimensional wave in a cavity. By the same basic physics of the Rubens tube, the flame table displays the modes via flame amplitudes based on the pressure nodes.

The Harold Daw flame table is described in this paper, a diagram of which can be seen in Fig.1. Detailed theory of the table is included as is a detailed description of the mechanical construction process. The intention for this table is that it will be used for outreach programs to garner an appreciation for science, mathematics, and engineering in future generations.

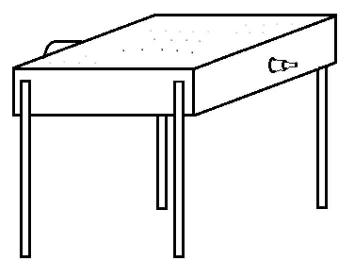

Figure 1: The rectangular two-dimensional flame table. The size of the table and the gas used to fill the cavity will affect which frequencies correspond to resonant sound modes.

Theory

The theory used to explain the workings of the two dimensional wave table rely upon conservation laws. We can begin the derivation for modelling different modes in a cavity by looking at conservation of mass. Using Euler's equations for fluid dynamics, conservation of mass can be expressed as

$$\frac{\partial P}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0 \tag{1}$$

where P is pressure, t is time, ρ is the density of the gas, and v is velocity [6]. Next, the momentum of the gas inside of the cavity needs to be dealt with. For this, Euler's equation for inviscid motion representing the momentum of a fluid is used [6]

$$\frac{\partial \rho \vec{v}}{\partial t} + \vec{\nabla}(\rho \vec{v} \cdot \vec{v}) = -\vec{\nabla} P \tag{2}$$

Next we need to take the divergence of Eq.2 and then combine it with Eq.1. Assuming that due to v^2 being so small the second term of Eq.2 goes to zero, we end up with the following relationship

$$\frac{\partial^2 P}{\partial t^2} = \vec{\nabla}^2 P \tag{3}$$

Now we must define some variables so as to make the problem simpler. First, we will define the number density as

$$\rho = \frac{N}{V} \tag{4}$$

where N is equal to the number of moles multiplied by the number of molecules and V is the volume. Let us also note that the pressure and the density will both experience small changes throughout the experiment. We can express the pressure and density as follows

$$P = P_o + \partial P, \quad \rho = \rho_o + \partial \rho \tag{5}$$

where P_o is the initial pressure of the gas and ρ_o is the initial density of the gas. These relations will appear later in the paper. Next, we must draw our attention to the Ideal Gas Law. Written in its most elegant form, the ideal gas law is expressed as

$$PV = Nk_bT \tag{6}$$

where T is the temperature of the gas and k_b is Plank's constant. Using Eq.4 we can write this in a form more convenient for our use

$$P = \rho k_B T \tag{7}$$

Now that we have a convenient form of the Ideal Gas Law, we need to look at the energy of the system. From conservation of energy and the First Law of Thermodynamics, we know that

$$dU = dQ + dW \tag{8}$$

where U is the total energy, Q is the heat energy, and W is the work. We know that our system is adiabatic via the entropy of the system, hence $Q = 0$. We also know that work can be written as the negative product of the pressure and the volume. Knowing this, Eq.8 can be re-written as

$$dU = -PdV \tag{9}$$

Again we must look at the total energy of the system, though this time from the Ideal Gas Law. The total energy of a system for an ideal gas can be expressed in two forms as

$$U = \frac{3}{2}k_bT = \frac{3}{2}PV \tag{10}$$

After combining the above equation with Eq.9 we can reduce it down to the following form

$$\frac{3}{5}\frac{1}{P}dP = -\frac{1}{V}dV \tag{11}$$

By differentiating Eq.4 and assuming that we are working with only one particle, we can re-write Eq.11 as

$$\frac{3}{5}\frac{\rho}{P} = \frac{d\rho}{dP} \tag{12}$$

Now, we can recall back to our note about the initial pressure and density of the gas from Eq.5. Assuming that the small changes in the pressure and density are small, we come to the expression

$$\frac{3}{5}\frac{\rho_o}{P_o}dP = d\rho \tag{13}$$

From Eq.2 we notice the following has to be true

$$\frac{3}{5}\frac{\rho_o}{P_o} = \frac{1}{v^2} \tag{14}$$

which can be interpreted as the speed of sound through a chosen gas [5]. Therefore we can formulate the following partial differential equation

$$\frac{1}{v^2}\left(\frac{\partial^2 P}{\partial t^2}\right) = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \tag{15}$$

By noting that the T(t) term relies on normal modes [1], we end up with the following separable O.D.E.

$$\frac{-1}{v^2}XY\omega^2 e^{i\omega t} = X''Ye^{i\omega t} + Y''Xe^{i\omega t} \tag{16}$$

The boundary conditions for the resultant solutions to the separated O.D.E.s are determined by the length of the box. According to Bernoulli's equation of energy conservation, the pressure will equal zero at the edges of the box because the velocity is zero at these bounds. From our boundary conditions, we end up with the following solution to the P.D.E.

$$P = P_o \cos\frac{m\pi x}{l} \cos\frac{n\pi y}{l} \tag{17}$$

where l is the length of the box, and x & y are horizontal and vertical coordinates on the top plane of the box, respectively. This solution can be plotted in Mathematica for various values of m & n. The values of m & n are determined by the following equation

$$f = \frac{v\sqrt{m^2+n^2}}{2l} \tag{18}$$

where f is the desired frequency and v is the speed of sound of a chosen gas. Alternatively, we can find the frequencies of various modes by simply plugging in values of m & n.

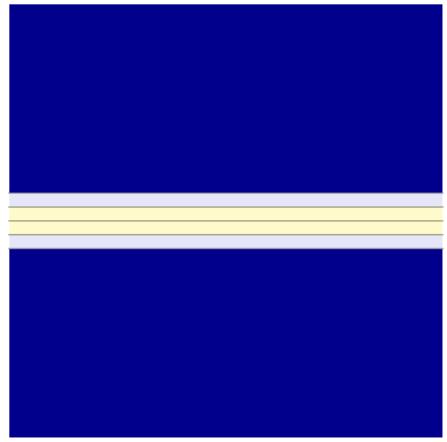

Figure 2: Normal mode m=0, n=1 of a gas in a 21" x 21" x 3.25" flame table. The expected frequency of this mode is f = 94Hz.

Application of Theory

The 2-D dimensions of our box are 0.5334m x 0.5334m (21" x 21"). For our experiment, we chose to use first methane gas, then propane gas to see how the frequencies differ. Methane has a density of 0.655 $\frac{kg}{m^3}$ which, according to theory, should produce modes at fairly normal frequencies (100 - 1000 Hz). Propane has a much higher density, at 1.879$\frac{kg}{m^3}$, and should hence produce modes at even lower frequencies. Expected images of the normal modes can be seen in Fig. 2 - 5.
It is also important to note that the uncertainty in the frequency calculation can be determined by the following equation

$$\Delta f^2 = (\frac{\partial f}{\partial P})^2 \Delta P^2 + (\frac{\partial f}{\partial l})^2 \Delta l^2 + (\frac{\partial f}{\partial \rho})^2 \Delta \rho^2 \tag{19}$$

For methane, the expected uncertainty of frequency for our box is ± 32.5 Hz, assuming that the uncertainty in gas density is very low.

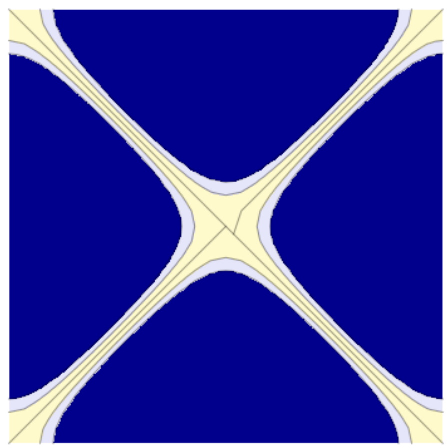

Figure 3: Normal mode m=2, n=0 of a gas in a 21" x 21" x 3.25" flame table. The expected frequency of this mode is f = 188Hz.

Mechanical Apparatus

The Daw flame table for this experiment was constructed by L. Heffern. The table is made of 1/8" cold rolled steel sheet metal with dimensions 21" by 21" by 3.25" with a speaker mount hole and a gas port on opposite sides, as seen in Fig.1. The top of the box consists of 400 1/8" diameter holes spaced on a 1" grid system. The holes of the box were made using a CNC machine which was operated by a manufacturing student at CSUC, Cody Leuck. The specifications for this flame table were

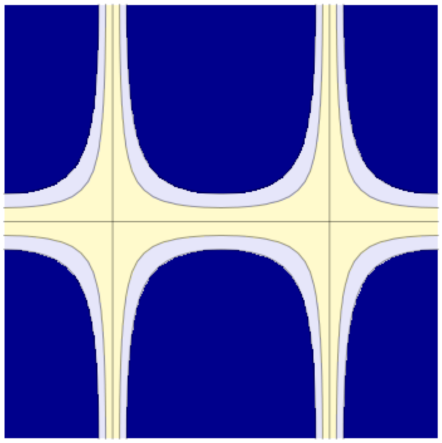

Figure 4: Normal mode m=2, n=1 of a gas in a 21" x 21" x 3.25" flame table. The expected frequency of this mode is f = 210z.

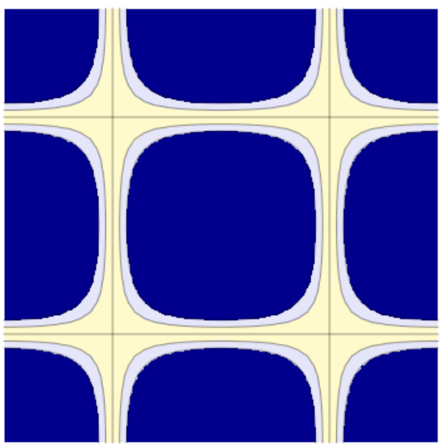

Figure 5: Normal mode m=2, n=2 of a gas in a 21" x 21" x 3.25" flame table. The expected frequency of this mode is f = 265Hz.

modified from the Daw's original design [3].

The welding of the box was done in several parts. First, a 3" diameter hole was cut into one of the 21" by 3.25" steel pieces. Next, a 3" diameter by 3" long steel cylinder was welded onto the hole using a MIG welding machine. This cylinder will act as a buffer between the flame table and the speaker.

On another 21" by 3.25" steel piece, a hole for the gas port was cut and a gas nozzle was mounted and brazed into place using a TIG welder. Next, the bottom of the box was welded together using 90° magnets and C-clamps so as to make sure the box is air-tight. Finally, the leg pieces (which were scrap metal) and the top, most important piece of the box were welded into place using a MIG welder. Throughout the process, there was minimal heat damage. Also, it is important to note that the legs should all be level so that the gas escapes in an equal distribution out of the table.

A gas hose was attached to the gas nozzle and then hooked up to two methane gas lines via a T-connection (to double the pressure). The speaker we chose to use is a basic computer speaker with a maximum of 5W power output; with high pressure this speaker works well enough to drive the air and produce a single mode. The speaker is hooked up to a frequency generator which then produces the desired frequencies.

Operation of Apparatus

The flame table can be connected to either a high pressure gas line or a standard propane tank. After allowing gas into the cavity it is important to take caution when lighting the table; it is important to know about the density of the gas in use. For example, methane will light above the table because it rises easily, whereas propane will light at the edges of the table because it tends to settle down the sides, rather than rise. [4]

Discussion

After testing the flame table, we determined that there are several factors that will increase its function to produce patterns of resonant modes. Upon initial testing, we determined that the table top was uneven due to heat damage, the speaker being used was not large enough to drive the gas, the hole size was inadequate, and the gas pressure was not high enough. We also determined that the speaker placement could only resonate the m=0 , n=1 mode due to symmetry of the box.

Upon testing the table with propane we observed that the flames moved upon the table in a chaotic path. The propane was used at a pressure much higher than that of the methane, though clearly the hole diameter was allowing too much gas to escape. The hole diameter caused the table to appear as though it was a "campfire."

The most convenient path of improving the box would be to add two extra speakers on the same side as the first speaker, to level the table via careful heat warping using an oxy-acetylene torch, and to add a second gas port. Ultimately, the most desired improvement would be to simply replace the top piece of the table to allow for a piece with smaller diameter holes. The gas escape from the larger holes causes the flame distribution to be dramatically uneven.

Speculations

Upon observing the flame table more closely, we observed fluctuations in the flames. In some instances the table appeared as though it were "alive and breathing." Further investigation as to the exact cause of these fluctuations would be an interesting though complex experiment in itself.

As noted by Daw, it would be fascinating to observe the flame table phenomenon of a violin-shaped cavity [4]. However, getting the rectangular cavity to display modes should be a requirement before attempting to build a violin-shaped one. Upon further speculation, the theory for such a cavity would likely be solved using numerical methods.

Acknowledgement

I would like to conclude this paper with several acknowledgements to the various people who contributed. First, I would like to thank Dr. Luis Buchholtz for his much appreciated help on the theory portion of this paper. I would also like to again recognize and thank Cody Leuck for programming and running the CNC used to drill the 400 holes. Matt Aaron, another CSUC manufacturing student, was especially helpful in the final welding portion of this project as an extra hand. I would also like to thank Dr. David Kagan for yelling at us to use a function generator instead of a computer. Ben Aguirre (Dana's significant other) was also helpful in operating the table during testing. Finally, I would like to thank my father, James Heffern, for the use of his machine shop, for his help in the initial welding and design process, and for teaching me long ago the skills necessary to build such an apparatus.

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3.2 Harold Daw's Flame Table FEA

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Introduction and Background

The Harold Daw flame table is a physics demonstration piece that helps students to visualize the two-dimensional wave equation based on pressure. The apparatus consists of a hollow cavity with holes on the top that allow gas to escape, inlet(s) that allow a forced frequency to be emitted, and gas inlet(s) to supply a flammable gas. When a frequency is put through a sound inlet of the apparatus, the gas inside the cavity changes pressure based on the wave equation (frequency is proportional to velocity, which is in turn proportional to pressure). When the holes on top of the apparatus are ignited, the flames contour along the top of the table, allowing students to view different patterns based on frequency. These patterns describe the Eigen modes of the two-dimensional wave equation.[1]

In order to perfect the construction of the apparatus, it is helpful to determine the stresses within the cavity. By determining the stresses along the sides of the cavity, we can determine which materials and material thicknesses will displace the least amount. Determining proper materials can lead to minimizing the cost of the apparatus. Determining the temperature at the exposed edge of the nozzle is also important in order to determine which types of tubes will be appropriate to attach. The flames on the top of the table create a temperature gradient based on different patterns. This temperature gradient can affect the temperature of the gas inlet and possibly melt or ignite the inlet tubing that attaches over the gas inlet. The temperature at the sound inlet also needs to be determined for a similar purpose: to avoid melting the speaker. By applying thermal boundary conditions we can determine the temperatures we are looking for. The lengths of the table legs can also be shortened based on knowledge of the temperature along different nodes of the legs. By determining when the table leg reaches either ambient air temperature or a “safe” temperature (of which it won’t melt what it’s standing on) we can cut the legs to save on material cost.

[1] I based this project off of my senior physics project: Harold Daw Flame Table. The paper for that project is available here. I wanted to know the information from this simulation in order to optimize the design of my own table.

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Modeling Assumptions

- The pressure inside of the cavity along the sides, top, and bottom will be assumed to be a maximum of 0.00015 psi. This value is based on a mean pressure gauge reading of 3.0 psi from a 1/4” methane outlet in the physical sciences building. The interior area of the cavity was compared to the methane outlet in order to determine an approximate maximum pressure within the cavity.
- The fixed point boundary conditions for the static study will be based on the feet of the apparatus.
- For the thermal study, a flame temperature of 1850 ^{o}C (average flame temperature of methane combustion with air) per flame outlet will be applied to each hole edge. *Note that this will give us a steady state temperature distribution* so any design changes made due to analysis results will be conservative.
- A convection coefficient of 3.0 W/M^{2}K with an ambient temperature of 25^{o}C will be applied to exterior surfaces. These numbers are based on realistic numbers from the heat transfer book used in Mech 338.
- A convection coefficient of 5.0 W/M^{2}K with an ambient temperature of 25 ^{o}C will be applied to interior surfaces, assuming that though the gas density increases, the overall convection increases due to increased gas velocity.
- The gas inlet nozzle to the table was simplified from the standard “stepped” geometry with threading, to a simple extruded solid cylinder. This simplification is conservative as the stepped geometry, hollow interior, and threads would increase surface area, which in turn would increase heat transfer by convection (cooling is underestimated).
- The number of holes in the table top was reduced from 400 (20x20) to 100 (10x10) in order to allow meshing (the memory on the computers in OCNL is lacking). The hole spacing was also changed from 1 inch to 2 inches, center to center spacing. The hole diameter was also doubled from 1/16” to 1/8”. These simplifications will have a negligible effect on the temperature gradient.
- It is important to note that in reality the table itself is entirely welded together, except in regards to the gas inlets which were braised. Hence, coincident mates were kept as they already simulate welds/braising.
- Mesh controls on the top of the table, as well as around the gas inlet were required for proper meshing.

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Results

Model Properties

With exception to the brass gas inlet nozzles, the apparatus is made of 1/8" cold-rolled ANSI 1020 steel. The top and bottom sheets are made of 21" x 21" pieces, whereas the side pieces are cut to 20.875" x 3.5" and nested into the sides by coincident mates.

Values of Interest

Static Study

- *Maximum stress within cavity and its location:* 1.18 psi located just above the sound ports on the top piece of the table.
- *Stress at the bottom center of the cavity:* 0.067 psi
- *Maximum displacement within cavity and its location:* 7.381 µ-in at the bottom center of the cavity

Thermal Study

For the thermal study it is important to note that the melting point of Teflon (which will be used for the gas tubing) is 327^{o}C and the melting point of acrylonitrile butadiene styrene (which we will assume is what encases the speaker) is 110^{o}C.

- *Temperature at the end of the gas inlet nozzle:* 904.78 ^{o}C
- *Temperature at the outside end of the sound inlet:* 690.93 ^{o}C
- *Temperatures up one leg of table starting from the floor (*^{o}C*):*

Location	0" (floor)	4.25"	6.18"	8.94"	13.701"	16.10"	18.51"	20.35"
Temperature (^{o}C)	220.84	229.19	254.60	342.22	406.80	492.24	598.95	736.00

Static & Thermal Combined Study

- *Maximum stress within cavity and its location:* 264.63 ksi located on the inside edge of a side connecting the sound port to the table, see Fig. 4.
- *Maximum displacement within cavity and its location:* 0.455 in located at the bottom center of the cavity

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Contour Plots

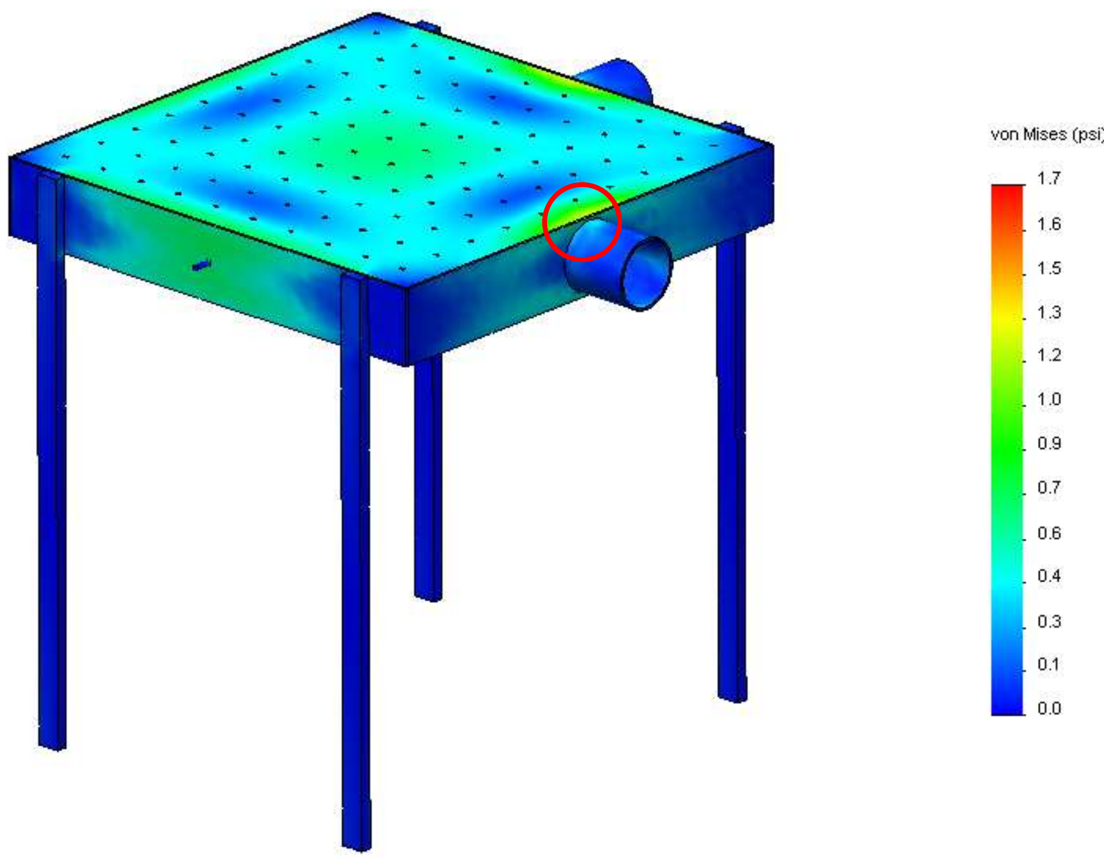

Figure 1: Contour plot of stress concentrations

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Contour Plots Cont.

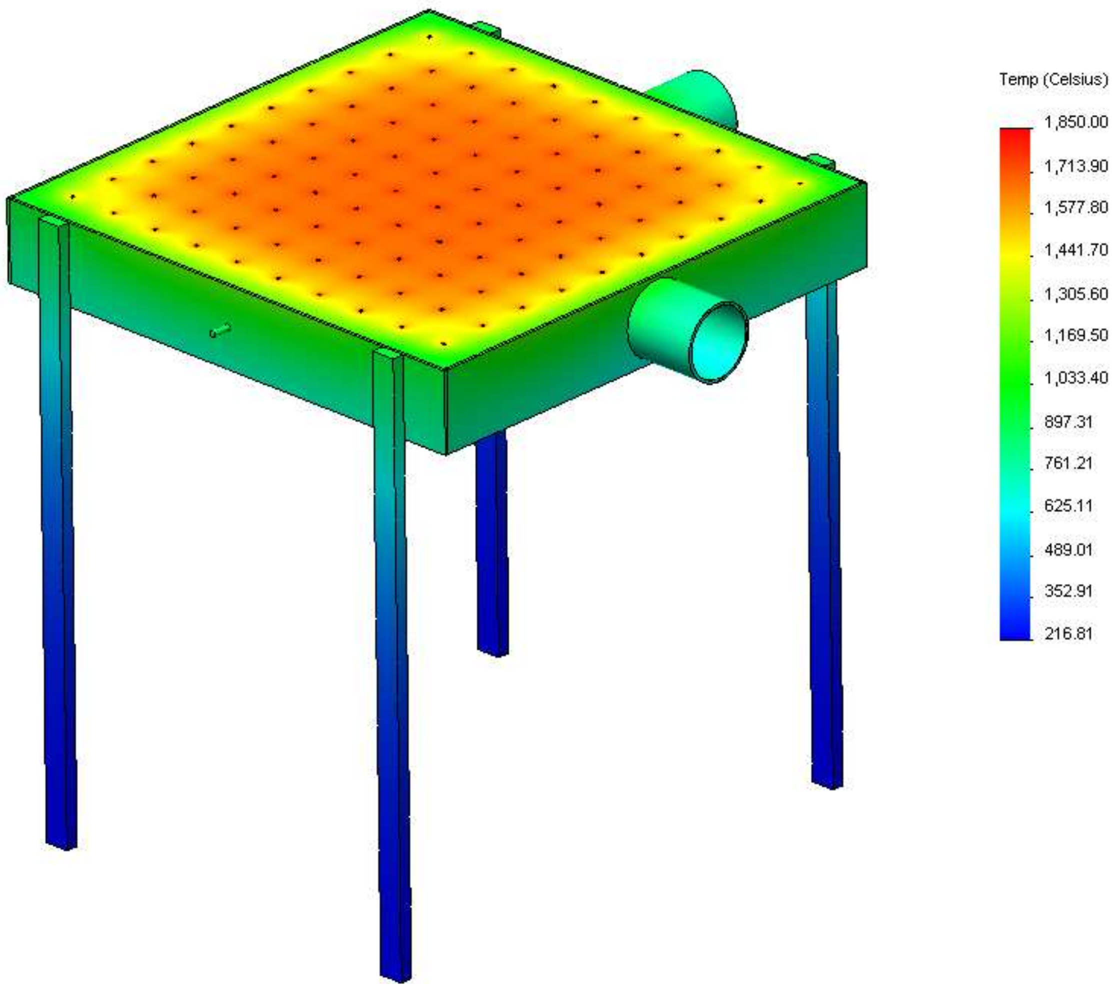

Figure 2: Contour plot of thermal effects

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Contour Plots Cont.

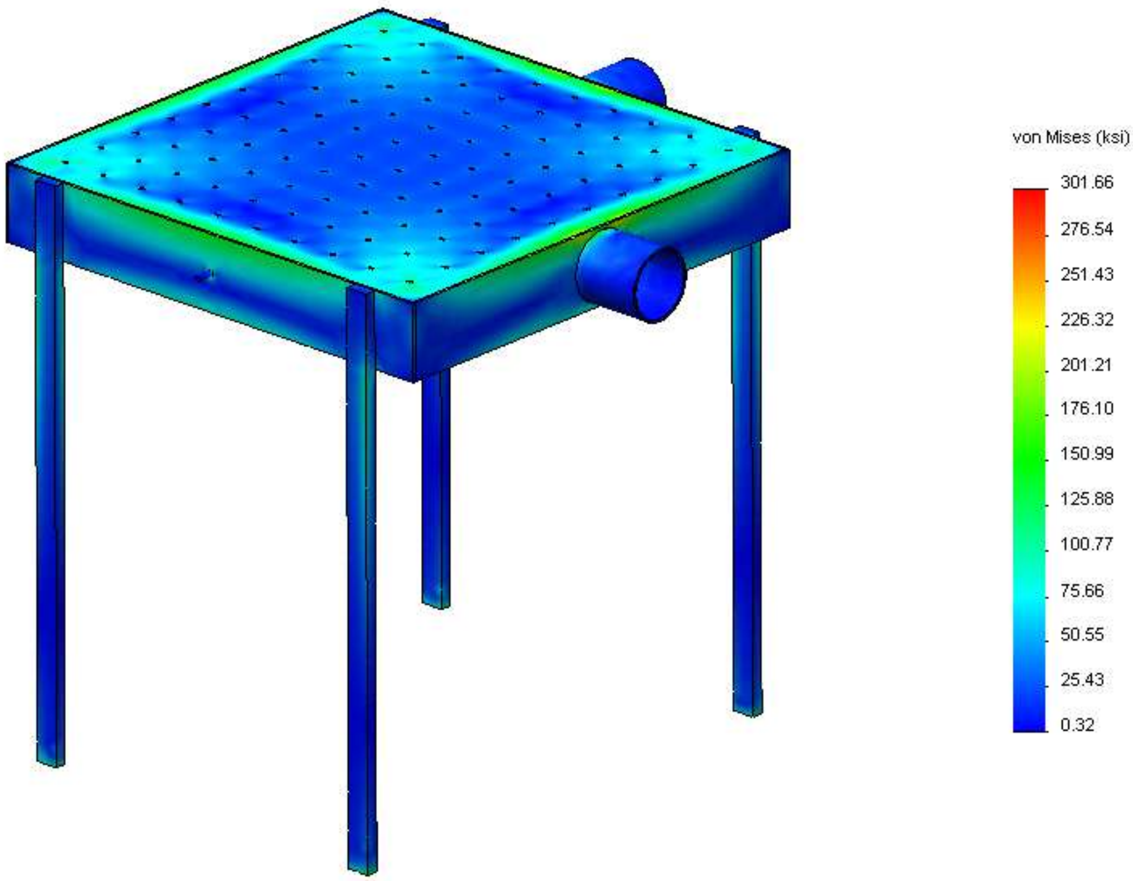

Figure 3: Contour plot of thermal stress concentrations

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Contour Plots Cont.

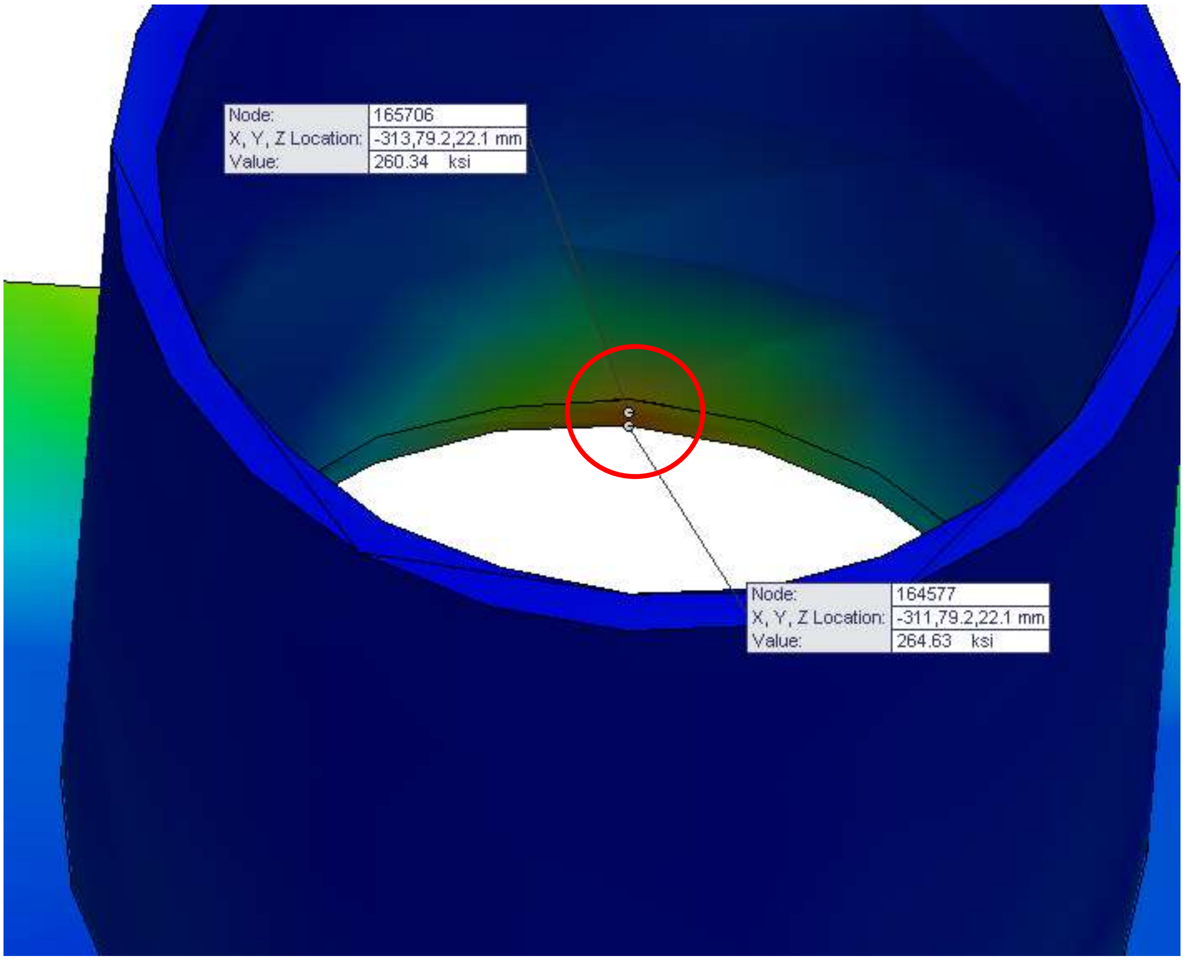

Figure 4: Maximum Thermal stress concentration

Temperature and Pressure analysis of the Harold A. Daw Flame Table

Conclusions

Determining the temperature distribution for such a complex geometry is only possible through numerical techniques, hence why SW Simulation was used. The FEA showed that my choice of brass for the nozzle was a good initial decision as the brass isn't a very good thermal conductor. However, to be sure of this assumption, an FEA was required because we're dealing with such high temperatures.

The FEA showed that thinner steel can be used in place of the 1/8" thick cold-rolled steel, as the static displacements were less than 0.005 in. Though when thermal stress was analyzed it could be seen that the displacements maxed out at about 0.5", though this is due more to thermal expansion than to pressures on the cavity walls.

The analysis also showed that the nozzle tip temperature (904.78^{o}C) greatly exceeded the temperature at which the polytetrafluoroethylene (Teflon) piping hits its melting point (327 ^{o}C). By moving the nozzle placement to the bottom of the cavity, we can avoid the higher temperatures and allow the nozzle tip to not melt the plastic.

The legs of the apparatus are at an appropriate height to avoid damaging any surfaces. Presumably we will be using this device on concrete and tile, so a temperature of 220^{o}C is safe enough to avoid damaging surfaces. The sound inlet edge temperature is well beyond most the melting point of ABS plastic (110 ^{o}C). To avoid melting, we can increase the length of the sound inlet tube or we can move the sound inlet tube to the bottom of the cavity, though this may cause undesired physical effects (strange Eigen modes, etc.).

Finally, I would like to note that in regards to the thermal FEA, all heat transfer was computed assuming a steady state. In reality, the table itself will only be able to run for as long as the propane tank can stay full. It is unlikely that the table will be run for more than 10 minutes at a time. If I were to design the flame table so that it could run indefinitely, then I would implement all of the suggested design changes. However, from actual experiment I know that only the Teflon tubing will begin melting within 10 minutes.